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ABSTRACT

This study investigated several aspects of a teaching experiment which focused on five seventh grade students' performance on nonroutine number theory problem-solving with a close look at the role of heuristics, concrete models, and the relationships between these variables. In the experiment, students were given a series of lessons on number theory, heuristics, concrete models, and the microcomputer. The microcomputer was used to present the problem, facilitate hint selection through the use of a menu, and to record student work. Interest was in identifying student behaviors after the experiment, not on identifying cause-effect relationships or predicting future behaviors. Three questions were addressed: (1) Is there an observable pattern of self-directed use of concrete models? (2) Will students who choose to use the provided concrete models improve as problem solvers? and (3) What motivates these students either to use or not to use the provided concrete models? Results indicate that all students showed improvement in problem-solving ability and increased use of concrete models. Compared to above-average and below-average students, the average ability students showed the greatest gain in demonstrated problem-solving ability and the greatest use of concrete models. (Author/JN)

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The Average Ability Middle School Student and Concrete Models in Problem Solving: a Look at Self-Direction

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Abstract

This is based on a teaching experiment focused on seventh graders' performance on nonroutine number theoretic problem solving with a close look at the role of heuristics, concrete models, and the relationships between them. Subjects were given a series of lessons on number theory, heuristics, concrete models, and the microcomputer. The microcomputer was used to present the problem, facilitate hint selection through the use of a menu, and to record student work. This paper is a report of self-directed use of concrete models by five average ability students who participated in the study.

All students showed improvement in problem-solving ability and increased use of concrete models. Compared to above-average and below-average ability students, the average ability students showed the greatest gain in demonstrated problem-solving ability and the greatest use of concrete models.

The Average Ability Middle School Student and Concrete Models in Problem Solving: A Look at Self-Direction

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The use of concrete models in the middle school is tricky business. Mathematics teachers tend to stop using them long before they should, which means that it has probably been years since any given middle school student has seen or used any. We know that mathematics teachers teach more than content; they also implicitly teach style of solving problems, attitudes, confidence in resources, etc. Children are, therefore, "taught" that concrete models are childish and unnecessary. Worse still, children are unable to make appropriate value judgements about these matters since they view teachers as models of correct problem-solving behaviors (Lesh & Schultz, 1983). Impressions and misconceptions about the use and misuse of concrete models are more critical in the elementary school than the secondary school, "for the damage done by the adoption of objectionable teaching strategies at the elementary level is usually ineradicable..." (Hilton, 1984, p.7). As a result, the widespread absence or misuse of concrete models in the middle school leaves children with misconceptions about the teaching and learning of mathematics in general and problem solving in particular.

This paper reports some aspects of a teaching experiment which focused on seventh graders' performance on nonroutine number theory problem solving

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with a close look at the role of heuristics, concrete models, and the relationships between them. The focus here is to report on self-directed use of concrete models by the average ability students while problem solving after being taught manipulative strategies "with the same degree of seriousness we would devote to any other mathematical technique" (Schoenfeld, 1980, p.17). Interest is in descriptions of student behaviors after this instruction, not on identifying cause-effect relationships or predicting future behaviors (Shumway, 1983). The following questions are addressed in this paper: (a) Is there an observable pattern of self-directed use of concrete models? (b) Will students who choose to use provided concrete models improve as problem solvers? and (c) What motivates these students either to use or not to use provided concrete models?

Perspective

Self-Direction. Two factors motivated the decision to look at the self-directed use of manipulatives. (1) the nonmanipulative approach is so ingrained in these children that to require use of manipulatives might have violated their own self-respect. Pilot work revealed strong resistance to manipulative models such as dice, counters, number tiles, and other miscellaneous items. In general, it was the opinion of the students that these materials were not needed. (2) I share with others (Sigel, Brodzinsky, & Golinkoff, 1981) Piaget's assumption that knowledge is actively constructed and applied through the knower's self-regulated interaction with the environment.

Concrete Models. Two factors enhance the argument to use concrete models. The first factor regards the fact that middle school students are

still learning the basics, while the second regards the relationship between problem solving and learning the basics. These factors are elaborated below.

(1) Though seventh graders have been taught essentially all elementary school mathematics, they are still refining their understandings of concepts, principles, and algorithms and will continue to do so for some time to come; moreover, they are still skill building. Regretfully, children, who are in transition from the concrete operational to the formal operational stage of cognitive development, are seldom, if ever, provided concrete models to assist in the process of refining their mathematical ideas (Schultz, 1979). Post's (1980) description of concrete models as more abstract than an actual situation, but less abstract than formal symbols, supports the use of these aids by the problem solver faced with having to process a problem. Because, he reminds us, an important position in mathematics is that if any two systems can be shown to be isomorphic to one another, it becomes possible and desirable to work in the simpler or more available system and transfer all conclusions to the less accessible one. Unfortunately, a common mistaken belief is that manipulatives are more useful in the primary grades than in the middle or secondary grades.

(2) Two different but related questions come up about basic skills, which I believe relate to the issue of concrete models in the classroom. The first is an "either/or" question, and the second concerns "how much." First, we know that there is a tremendous emphasis on computational drill, which may be counterproductive to the development of the flexibility of thinking essential in problem solving. Kantowski (1981) asks "how can this

demand for emphasis on basic skills be reconciled with the need for development of problem-solving ability in the limited time available for mathematics instruction" (p.121)? The second is Kilpatrick's (1984) question about meaning and automaticity: "For each skill in the school mathematics curriculum, what level of automaticity is optimal for subsequent use [emphasis mine] of a skill, and can the skill be made meaningful without inhibiting automaticity" (p.45)?

One way to approach basic skills and problem solving in the middle school is to support the notion of nurturing the continuing development of concepts and skills through the problem-solving process where manipulative models are available. The problem-solving process for some individuals some of the time is a matter of integrating correct, stable mathematical understandings. And for others, perhaps for many individuals most of the time, problem-solving solutions are constructed by slowly and thoughtfully organizing, integrating, differentiating, correcting, and stabilizing mathematical understandings (Brown, 1984; Lesh, 1982; Lesh & Schultz, 1983; Webb, 1979). Therefore, problem solving by nature for many means a continued learning of basic mathematical ideas and development of skills. Of course, we should familiarize students with a substantial amount of mathematics and equip them with basic skills before involving them in problem solving, but "'coming to know' does share some important elements with the activity of problem solving" (Brown, 1984, p.9). And if one is still in the process of coming to know, then the use of concrete models most surely must be facilitative for those in the transition between the concrete and formal operational stages of cognitive development.

Average Ability Students. Again, there are two factors to consider. (1) Another misconception about using concrete manipulatives is that they are more suited for below-average students than average or even above-average students. Historically, Title I classrooms were equipped with an abundance of Cuisenaire rods, chip trading materials, bundled sticks, geoboards, etc., whereas there continued to be a dearth of such items in the "regular" classrooms.

(2) As would be expected, research has shown that above-average or gifted students, in general, have been found to be better problem solvers (Hart, in progress). Knowing this, research on the above-average student has been done to identify the techniques characteristic of good problem solvers so these techniques could be taught to the average and below-average students. But, the processes used by above-average students might be unavailable to the average student (Dalton, 1974; Knippenburg, 1978; Lesh, 1981; Lester, 1982). For example, good and poor problem solvers differ in what and how they recall information from problems. Good problem solvers tend to forget details and recall structural characteristics, while poor problem solvers tend to recall specifics (Silver, 1981).

Research which focuses on the problem-solving techniques of the average ability student is relatively recent and generally hypothesis generating. In fact, to date the research of Barnes & Todd (1977), Lesh (1982), and Noddings (1983) are the only significant works which have addressed this issue (Hart, in progress).

Method

The project, which took the entire school year, included all three seventh grade classes of a suburban middle-class Atlanta public school. In all there were 30 subjects out of 78 seventh graders. The focus of this report is on five subjects determined to be average. That is, these students did not qualify for a remedial or gifted program in the school system, they functioned within 18 months of grade level based on results of the California Achievement Test, and they demonstrated average problem-solving ability based on a weighted score of solution correctness with respect to the other students tested on a Cold Problem-Solving Test (described below).

Students were given a series of lessons on number theory, heuristics, concrete models, and the microcomputer. Concrete models consisted of such materials as dice, picture cards, number tiles, blocks, plastic cars, disposable cups, styrofoam pieces, pennies, bottle caps, etc. The microcomputer was used to present the problem, facilitate hint selection through the use of a menu, and to record student work. This paper summarizes some aspects of the study that concern self-directed use of concrete models throughout the phases of the project.

On the first day a General Questionnaire was administered and an orientation to the project was given. This was followed the next day with a Cold Problem-Solving Test (no prior instruction about problem solving) with concrete models available if students wanted them. Then there was a two-week instructional period in which all the basic mathematics content necessary to solve the problems on the Cold Problem-Solving Test was reviewed. (Corresponding problems on the Cold Problem-Solving Test, Pre

Problem-Solving Test, instructional phase, computer problems, and Post Problem-Solving Test were parallel in structure e.g., one problem type structure required the student to identify two or three numbers given the product and sum of the factors.) Also in the two-week instructional period Polya's (1957) four stages of problem solving and several heuristics were introduced. Provided materials and concrete models were used for all the problems during this teaching intervention. A Pre Problem-Solving Test with concrete models available for voluntary use was given immediately after the two-week instructional period, which marked the end of the first phase of the project.

The second phase occurred during the next eight months while the students individually worked nine problems. A problem followed by a menu of hints was presented one at a time on the computer. Concrete models were again made available at each of two computer stations in the classroom where each of the classes met. The materials were there for the students' voluntary use.

The last phase of the study consisted of a day of administering a Post Problem-Solving Test with concrete models available for voluntary use, a day of administering a Post General Questionnaire and an Attitude Questionnaire, and a half day of individual interviews with selected students.

Results and discussion

The two sources of data presented here, which were analyzed for observable patterns of self-directed use of concrete models, are tests and computer problem-solving self-reports. (Reference is made to Schultz and

Hart (1983) for another discussion on this work.) First, the testing data showed that the average ability group used concrete models more on the Cold Problem-Solving Test than on the Pre Problem-Solving Test, dropping from 35% to 15% use. All groups showed a marked increase in use of models from the Pre to the Post Problem-Solving Test. For example, the average group increased from 15% to 83%, whereas the below-average group increased from 55% to 71% and the above-average group increased from 15% to 67%. The highest frequency of use of the models was by the average group on the Post Problem-Solving Test. On the microcomputer problems, the average group recorded the highest frequency of 36% use of concrete models, followed by the above-average group's 18%, and then the below-average group's 14% use of concrete models.

Second, as a whole, all three groups showed improving problem-solving ability as determined by a weighted score of solution correctness. The percents of solution correctness for the the Cold, Pre, and Post Problem-Solving Tests for the average group were 40%-49%-62%, respectively. Increasing percents correspond to increasing use of the concrete models for all groups only when considering the Cold and the Post Problem-Solving Tests. Though the average and above-average groups performed increasingly better on the tests, they reduced their use of concrete manipulatives to 15% each on the Pre Problem-Solving Test--the middle test.

Third, it was reported in general that students did not prefer the models on the Cold and Pre Problem-Solving Tests and while working the computer problems because they were not sure how to use them, they became confused by them, or they did not see the need to use them. But while

taking the Post Problem-Solving Test, the students realized that all they had was the provided concrete models. They no longer had the computer with a host of hints to call on, some of which were animated illustrations of the problem situation. Students reported that they had to use the models for some problems, that the models were helpful, and some reported that they could not solve any problems without the models.

A closer look at the average students. Most interesting was the behavior of the average ability group, which used the models the most both on the Post Problem-Solving Test (i.e., 83% compared to 71% for the below-average and 67% for the above-average students) and on the computer problems as reported on their self reports (i.e., 36% compared to 14% for the below-average and 18% for the above-average students). The common mistaken notion that concrete models are for use only by low ability students is not supported by these results. These children who are of average ability and functioning on grade level were quite receptive and responsive to concrete models in problem solving.

The Post Attitude Questionnaire consisted of 19 statements about the project to which the students had to respond on a scale of 1 to 4, where 1 meant "none" and 5 meant "a lot." A sampling of these statements is below.

I enjoyed participating in the problem solving project.

The six classes we had at the beginning of the project helped me to learn new ways to approach solving problems.

If I had to solve problems without the microcomputer, I think that I would use the provided materials more often.

The pictures and diagrams provided were a help to me in solving the problems.

Participating in this project made me realize that using provided materials can be an important activity in solving problems.

A closer examination of the results on this instrument shows that the average ability group scored the highest (average score) on 14 items, tied with the above-average group on three items, tied with the below-average group on one item, and had the middle score on one item. That is, the average ability group had the highest or tied with the highest scores on all but one item. Ironically, the one item on which the average students had the middle score was "I used the provided materials to solve many of the problems." According to their standards of performance and expectations of themselves, the average ability group viewed their use of the manipulative materials as minimal, when comparitively speaking, they in fact used the materials the most.

What is obvious from the above discussion is that the average-ability students in this project stood out as distinctly different from the above-average and below-average students. Ultimately, it is hoped that research will result in instructional strategies suitable for the average ability problem solver.

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